Waveform Relaxation Synthesis of Time-Domain Characteristic Model of Loaded Microstrip from FDTD Simulation

Qing-Xin Chu and Fung-Yuel Chang, *Fellow, IEEE*

***Abstract*—From the finite-difference time-domain (FDTD) simulation of response voltages and currents of a microstrip terminated by a step-pulse excitation voltage source and a resistor load, the time-domain characteristic model (TDCM) of the microstrip is synthesized by use of the waveform relaxation method, which is based on the iteration and deconvolution techniques. As an example, the extracted model is applied to simulate the responses of the microstrip loaded by a capacitor and a resistor in parallel. The results are compared with the direct FDTD simulation to validate the accuracy of the model.**

I. Introduction

A MICROSTRIP line is a common interconnect in very large-scale integration (VLSI). Today, the clock speed of signals in VLSI has become so high that the dispersion, radiation, and interaction with loads of the microstrip have to be taken into account in design and simulation of high-speed microstrip circuits. Therefore, a compatible model of the microstrip for high-speed circuits is needed as a computer-aided design and simulation tool. Since the microstrip in high-speed circuits may load nonlinear devices with time-dependent characteristics, a time-domain model is more suitable and straightforward.

In [1], the authors presented a synthesis technique of time-domain model of microstrip based on traveling-wave voltages and currents on it. In order to obtain traveling waves by finite-difference time-domain (FDTD), however, the end-absorbing boundary has to be far away from the observing plane (such as $100\Delta y$) to reduce the effect of absorbing boundary condition (ABC). And this model did not include the interaction with the loads.

In 1994, Chang presented the time-domain characteristic model (TDCM) of distributed-lumped networks and its waveform relaxation synthesis method [2]. In this letter, the method is expanded for the loaded microstrip as shown in Fig. 1(a). Like in [1], the FDTD [3]–[5] is adopted to simulate the electromagnetic fields in the microstrip with a step-pulse excitation and a resistor load to get the accurate terminal response voltages and currents. It is obvious that the interaction

Manuscript received January 17, 1997. This work was supported by Hong Kong UPGC Earmarked Grants CUHK RGC 294/94E.

Q.-X. Chu is with the Department of Electronic Engineering, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong, on leave from the Department of Microwave Engineering, Xidian University, Xi'an 710071, Shaanxi, P. R. China.

F.-Y. Chang is with the Department of Electronic Engineering, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong.

Publisher Item Identifier S 1051-8207(97)03855-5.

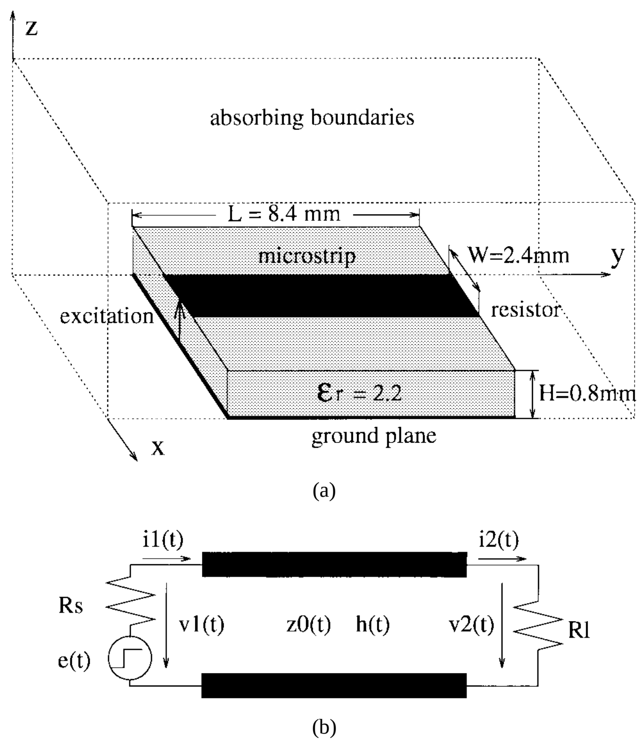

Fig. 1. Microstrip line (a) layout and (b) equivalent circuit.

of the microstrip with loads has been included in responses. Since the fields out of the microstrip attenuate rapidly, the ABC may be set up on where closer to ports (such as $10\Delta y$).

In order to demonstrate the accuracy of the model, the extracted model is used to simulate the responses of the microstrip loaded with a capacitor and a resistor in parallel. Comparison with the direct FDTD simulation shows a good agreement.

II. Time-Domain Characteristic Model

The layout of the microstrip under consideration is shown in Fig. 1(a). Fig. 1(b) shows its equivalent circuit. In frequency-domain, the response voltages V_1, V_2 and currents I_1, I_2 have the following relations:

$$V_j = V_j^+ + V_j^- \tag{1}$$

$$I_j = \frac{V_j^+ - V_j^-}{Z_0} \tag{2}$$

$$V_1^- = e^{-\beta l} V_2^- \tag{3}$$

$$V_2^+ = e^{-\beta l} V_1^+ \tag{4}$$

where Z_0, β are the characteristic impedance and propagation constant, respectively, superscripts +, − represent incident and reflected waves, respectively, and $j = 1, 2$ and l is length of the line. From (1) to (4), we get

$$V_1 - Z_0 I_1 = e^{-\beta l}(V_2 - Z_0 I_2) \tag{5}$$
$$V_2 + Z_0 I_2 = e^{-\beta l}(V_1 + Z_0 I_1) \tag{6}$$

which is known as the characteristic model of the microstrip. By using Gauss–Seidel type of iteration and letting the initial conditions be $(e^{-\beta l})^{(0)} = Z_0^{(0)} = 0$, we obtain the iteration equations for Z_0 and $e^{-\beta l}$ [2]

$$Z_0^{(k)} = \frac{V_1 - (e^{-\beta l})^{(k-1)}[V_2 - Z_0^{(k-1)} I_2]}{I_1} \tag{7}$$
$$(e^{-\beta l})^{(k)} = \frac{V_2 + Z_0^{(k)} I_2}{V_1 + Z_0^{(k)} I_1} \tag{8}$$

where k is iteration count.

Transformed into time domain,[1] (7) and (8) become

$$z_0^{(k)} = \frac{\{v_1 - h^{(k-1)} * [v_2 - z_0^{(k-1)} * i_2]\}}{\{i_1\}} \tag{9}$$
$$h^{(k)} = \frac{\{v_2 + z_0^{(k)} * i_2\}}{\{v_1 + z_0^{(k)} * i_1\}} \tag{10}$$

where z_0 and h are the inverse Laplace transform of Z_0 and $e^{-\beta l}$, respectively, which are known as transient characteristic impedance and transient delay function, respectively [1], and symbols $*$ and $\{\}/\{\}$ represent convolution and deconvolution operators, respectively. Equations (9) and (10) are the waveform relaxation equations for the synthesis of the TDCM of the microstrip.

Because the FDTD simulation is discrete, it is necessary to use the numerical convolution and deconvolution in (9) and (10). Consider a general form of convolution as

$$y(t) = p(t) * x(t) = \int_0^t x(t-u) p(u)\, du \tag{11}$$

and let $y(t) = x(t) = p(t) = 0$ when $t < 0$. Assume that the discrete values $y[n] = y(n\Delta t)$, $x[n] = x(n\Delta t)(n = 1, 2, \cdots, N)$ where Δt represents time step and adjacent values $x[n-1]$, $x[n]$ have a little difference. Then, (11) can be approximated as

$$y[n] = \sum_{k=1}^{n} x[n-k+1] p[k] \tag{12}$$

where

$$p[k] = \int_{(k-1)\Delta t}^{k\Delta t} p(u)\, du. \tag{13}$$

[1] Frequency- and time-domain functions are assigned by uppercase and lowercase letters, such as $\{V, I\}$ and $\{v, i\}$, respectively.

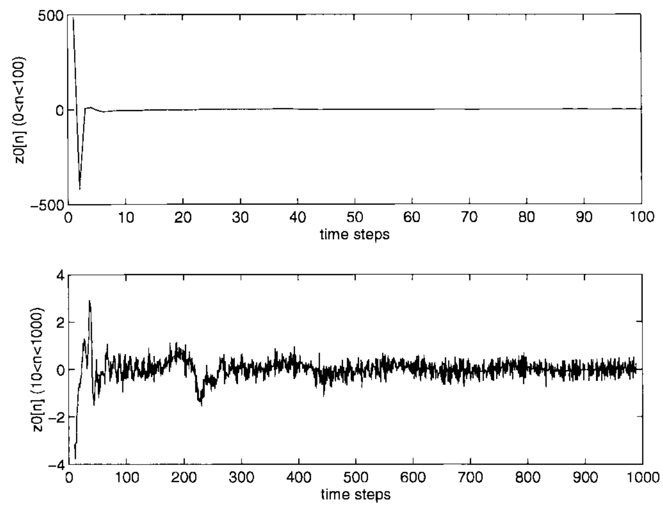

Fig. 2. Transient characteristic impedance of the microstrip.

From (12), the equation of numerical deconvolution is obtained

$$p[n] = \frac{\{y(t)\}}{\{x(t)\}} = \frac{y[n] - \sum_{k=1}^{n-1} x[n-k+1]p[k]}{x[1]}. \tag{14}$$

III. Numerical Results

First, the FDTD method is used to simulate the terminal response voltages and currents of the microstrip shown in Fig. 1(a). At one of terminal, a resistive voltage source with step-pulse excitation and internal resistor $R_s = 50\ \Omega$ bridges the strip and ground plane at the center of the line. A matched resistor $R_l = 50\ \Omega$ loads the another terminal in the same manner. The details of FDTD computation is the same with [1] except the end ABC's are set up on the plane away $10\Delta y$ from the ports and the simulation only need perform 1000 time steps, which takes about 3 min on Sunstation Ultra1. The response voltages are obtained from the line integral of the resulting vertical electric fields under the center of the strip at ports and the currents from circuit equations $i_1(t) = [e(t) - v_1(t)]/R_s$ and $i_2(t) = v_2(t)/R_l$.

Second, substitute these responses into (9) and (10). Through six iteration steps, we obtain $z_0[n]$ and $h[n]$ shown in Figs. 2 and 3, respectively, with 6 s on MATLAB. The mean-square error of iteration is less than 10^{-5}.

Finally, the extracted model is applied to simulate the terminal responses of the microstrip loaded by a capacitor $C = 2$ pF and a resistor $R_l = 200\ \Omega$ in parallel to demonstrate the efficiency and accuracy of this model. This process takes only 2 s on MATLAB as compared to direct FDTD simulation, which takes 180 s. Comparison of both results are shown in Figs. 4 and 5, which shows a good agreement.

IV. Conclusions

By means of the FDTD method and waveform relaxation technique, the TDCM of the microstrip terminated by an

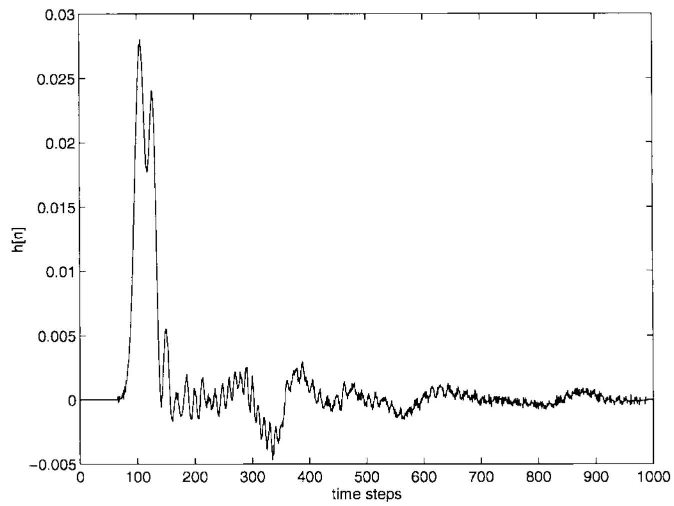

Fig. 3. Transient propagation function of the microstrip.

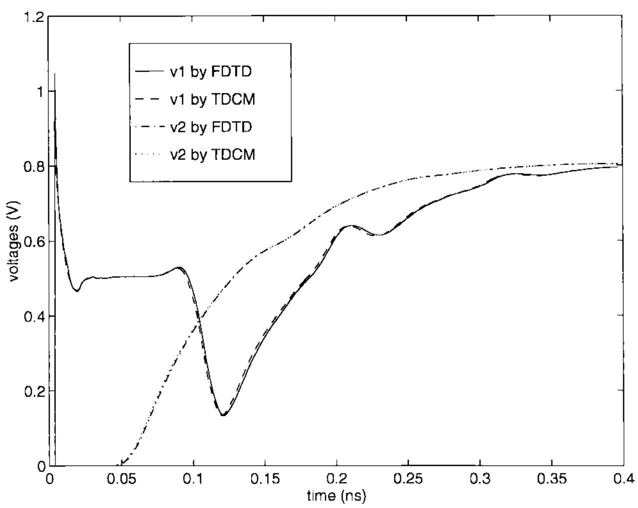

Fig. 4. Comparison of terminal response voltages of the microstrip terminated by a capacitor and a resistor in parallel by TDCM and by FDTD.

excitation source and a load has been synthesized. Since this model contains the electromagnetic details in the microstrip and its interaction with the source and load, it may occupy a wider frequency spectrum. The waveform relaxation technique based on iteration and deconvolution facilitates the synthesis of the model, and all transient functions of the model can be

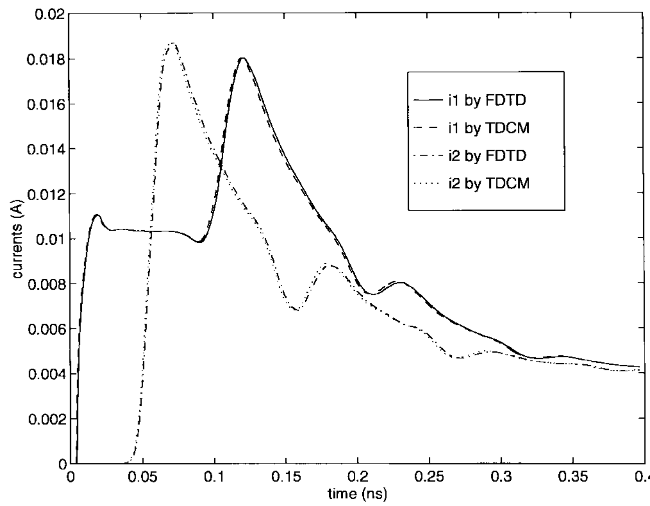

Fig. 5. Comparison of terminal response currents of the microstrip terminated by a capacitor and a resistor in parallel by TDCM and by FDTD.

obtained from the terminal responses in a single computational process with several iteration steps. With the model, responses of the microstrip with any load and excitation source can be rapidly obtained by use of numerical convolution and deconvolution. It has been proven in [2] that a two-port network can be characterized by characteristic model, so we believe that this procedure is also suitable for many microstrip interconnects such as steps, bends, vias, and various discontinuities.

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